Close Wednesday: HW_3A,3B,3C
(complete sooner!)
Office hours 1:30-3:00pm in PDL C-339
Exam 1 is Thursday in normal quiz section. Covers 4.9, 5.1-5.5, 6.1-6.3.

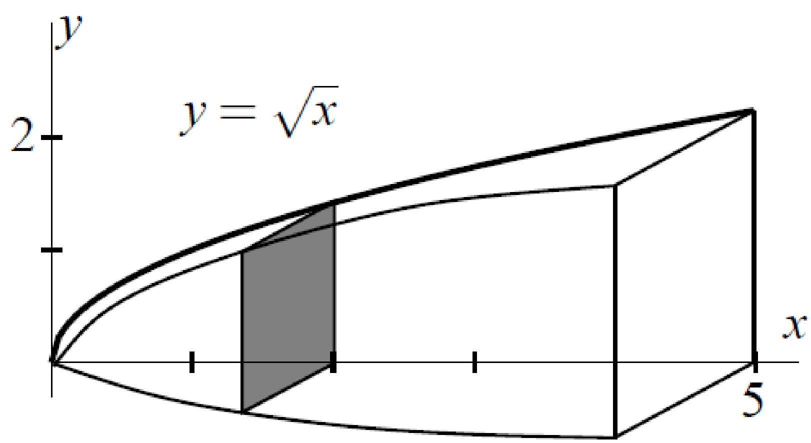
Entry Task:
Consider the region *R* bounded by $y = x^3$, $y = 8$, and $x = 0$.

Set up the integrals that would give the volume of the solid obtained by rotating *R* about the ….

(a) … *x*-axis.
(b) … *y*-axis.
(c) … vertical line *x* = -10.

1. Draw and label!
2. Cross-sectional area?
3. Integrate area.

Example:
(From an old final and homework)
Find the volume of the solid shown.
The cross-sections are squares coming out of the paper.



1. Draw and label!
2. Cross-sectional area?
3. Integrate area.

Example:

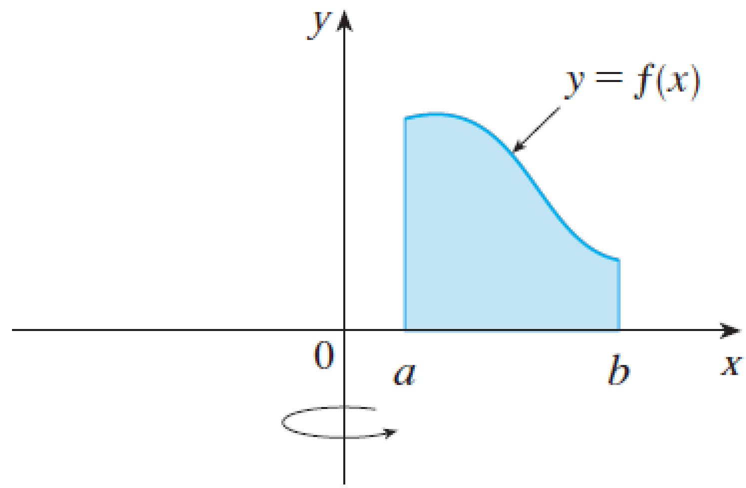
Let R be the region bounded by

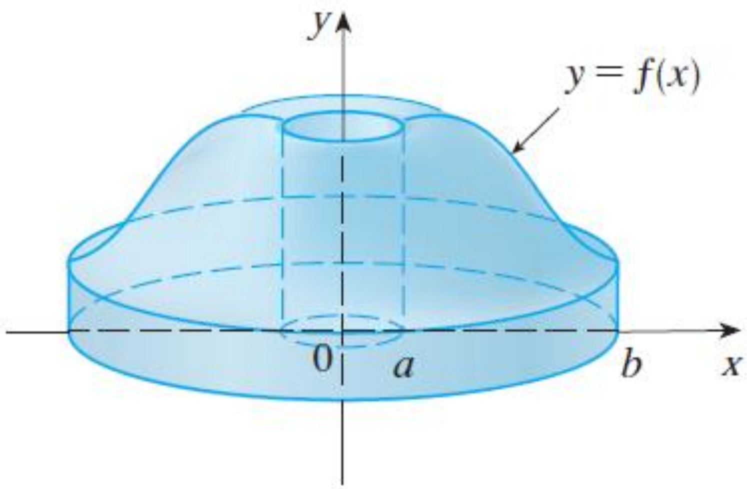
$$y = \frac{1}{x^2} + \frac{1}{x}, y = 0, x = 1, x = 2.$$

Try to use cross-sectional slicing to set up an integral for the volume obtained by rotating R about the ***y*-axis**. Why is this difficult/messy?

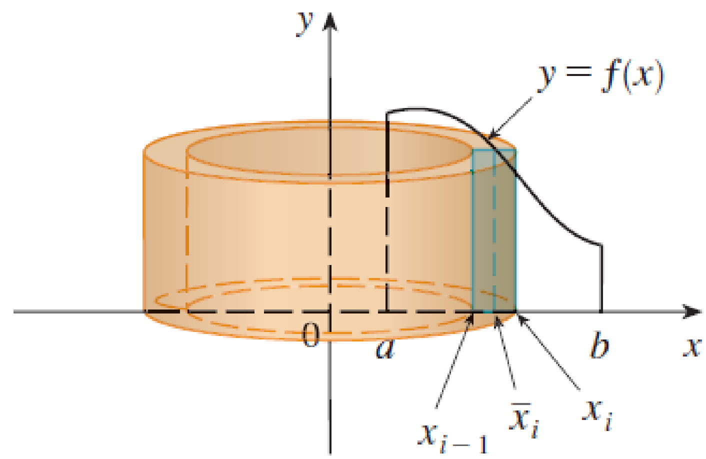
6.3 Volumes Using Cylindrical Shells
Visual Motivation:

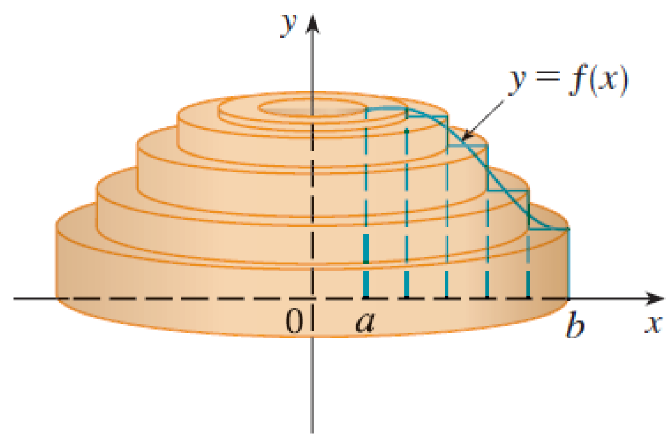
Consider the solid





We want to use “dx”, but that breaks the region into thin vertical subdivisions and rotating those gives a new shape, “cylindrical shells”





Derivation:

The pattern for the volume of one thin cylindrical shell is

VOLUME = (surface area)(thickness)

= $SA(x_i)\ \Delta x$

= 2π(radius)(height)(thickness)

$$\text{Volume} = \int_a^b SA(x)dx$$

$$= \int_a^b 2\pi(radius)(height)dx$$

Thus, if we can find a formula, $SA(x_i)$, for the surface area of a typical cylindrical shell, then

Thin Shell Volume ≈ $SA(x_i)\ \Delta x$,

$$\text{Total Volume} \approx \sum_{i=1}^{n} \text{SA}(\text{x}_i)\ \Delta\text{x}$$

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} \text{SA}(\text{x}_i)\Delta\text{x}$$

Example:

Let R be the region bounded by

$$y = \frac{1}{x^2} + \frac{1}{x}, y = 0, x = 1, x = 2.$$

Set up an integral for the volume obtained by rotating R about the ***y*-axis**.

Example:

Let *R* be the region in the first quadrant that is bounded by $x = \sqrt{y+1}$ and $y = 1$.

Find the volume obtained by rotating *R* about the ***x*-axis**.

Example:

Let R be the region bounded by

$$y = x^3, y = 4x,$$

$$\text{between } x = 1 \text{ and } x = 2.$$

1. Set up the integrals for the volume of the solid obtained by rotating R **about the y-axis**.
   (a) Using dy.
   (b) Using dx.

2. What changes if we rotate about the vertical line x = -2?

3. What changes if we rotate about the vertical line x = 3?

Flow chart of all Volume of Revolution Problems

***Step 0*: Draw an accurate picture!!! (Always draw a picture)**

Step 1: Choose and label the variable (based on the region and given equations)
If *x*, draw a typical **vertical** thin approximating rectangle at *x*.
If *y*, draw a typical **horizontal** thin approximating rectangle at *y*.

Step 2: Is the approximating rectangle perpendicular or parallel to the rotation axis?
Perpendicular → *Cross-sections*: Write

$$\text{Volume} = \int_a^b (\pi(\text{outer})^2 - \pi(\text{inner})^2)(\text{dx or dy})$$

Parallel → *Shells*: Write

$$\text{Volume} = \int_a^b 2\pi(\text{radius})(\text{height})(\text{dx or dy})$$

Step 3: Write everything in terms of the desired variable and fill in patterns.
Then integrate.

The above method is how you should approach problems, but if you are still having trouble seeing which variable goes with which method here is a summary:

Axis of rotation	**Disc/Washer**	**Shells**
x-axis (or any horizontal axis)	dx	dy
y-axis (or any vertical axis)	dy	dx